DBN-003-1163005 Seat No. ________

M. Sc. (Sem. III) Examination

June – 2022

Mathematics : EMT - 3011

(Differential Geometry)

Faculty Code : 003
Subject Code : 1163005

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Attempt any **five** questions from the following.

(2) There are total **ten** questions.

(3) Each question carries **equal** marks

1 Attempt the following : **14**

(1) Define with example: Regular curve.

(2) Define: ε- neighborhood in R^2.

(3) Find curvature and torsion of the circle $2x^2 + 2y^2 - 12x - 12y - 36 = 0$.

(4) Define with examples: Functions of class k.

(5) Define: Tangent vector field.

(6) Define: Length of a regular curve segment.

(7) Define: Unit speed curve.

2 Attempt the following : **14**

(1) Find the curvature and torsion of the curves

(i) $5x + 2y = 0$ and (ii) $x^2 + y^2 = 4$.

(2) Define with example: Simple surface.

(3) Define : The Osculating plane and the Rectifying plane. Also demonstrate them on a surface of an upper Hemisphere.

(4) Is the curve $a(x) = (x^{100}, 2x + 7, 5x^2 + 3)$ is regular? Justify your answer.

(5) Define: Normal curvature and Geodesic curvature.

(6) Identify the curve $x \cos \alpha + y \sin \alpha = p$ and find its curvature and torsion.

(7) Define: Velocity vector of a regular curve α.

3 Attempt the following : **14**

(a) Define tangent line to a curve. Show that the curve $\alpha(t) = (\sin 6t \cos t, \sin 6t \sin t, 0)$ is regular. Also find the equation of tangent line to α at the point $t = \frac{\pi}{6}$.

(b) Show that the curve $\alpha(S) = \left(\frac{5}{13}\cos S, \frac{8}{13} - \sin S, -\frac{12}{13}\cos S\right)$ is a unit speed curve. Also compute its curvature and torsion of the given curve.

4 Attempt the following : **14**

(a) Define reparametrization of a curve. If $g : [c, d] \to [a, b]$ is a reparametrization of a curve segment $\alpha : [a, b] \to R^3$ then prove that the length of α is equal to the length of $\beta = \alpha \circ g$. Also derive the relation between their tangent planes.

(b) Define the arc length of a curve and prove that the arc length is one - one function mapping (a, b) onto (c, d) and it is a reparametrization. Is the curve reparametrized by its arc length yield a unit speed curve? Justify your answer.

5 Attempt the following : **14**

(a) Let $\alpha(s)$ be a unit speed curve whose image lies on a sphere of radius r and centre m then show that $k \neq 0$. Also if $\tau \neq 0$ then $\alpha - m = -\rho N - \rho' \sigma \beta$ and $r^2 = \rho^2 (\rho' \sigma)^2$ (where $\rho = \frac{1}{k}$ and $\sigma = \frac{1}{\tau}$).

(b) Is the curve $\alpha(t) = (\sin t, \cos^2 t, \cos t)$ regular? If so then find the equation of tangent line at $t = \frac{\pi}{4}$.

6 Attempt the following : **14**

(a) Prove that: The set of all tangent vectors to a simple surface $x : u \to R^3$ at P is a vector space. Also find the dimension of that vector space.

(b) Show that the length of the curve

$\alpha(t) = \left(2\alpha\left(\sin^{-1} t + t\sqrt{1-t^2}\right), 2at^2, 4at\right)$ between the points $t = t_1$ to $t = t_2$ is $4a\sqrt{2}(t_2 - t_1)$. What will be the arc length between the points $t_1 = 25$ and $t_2 = 30$?

7 Attempt the following: **14**

(a) Define orthonormal vectors and prove that the set $\{T, N, B\}$ is orthonormal.

(b) Find the arc lengths of the curves $\alpha(t) = (r\cos t, r\sin t, 0)$ and $\alpha(t) = (r\cos \omega s, r\sin \omega s, h\omega s)$. Also reparametrize them by their arc lengths.

8 Attempt the following : **14**

(a) Show that a simple surface remains simple even after coordinate transformation.

(b) Prove in the usual notations the relation

$$g_{ij} = \Sigma g_{\alpha\beta} \frac{\partial v^{\alpha}}{\partial u^{i}} \frac{\partial v^{\beta}}{\partial u^{j}}.$$

9 Attempt the following : **14**

(a) Define Monge patch and compute coefficients of first and second fundamental form. Also find Christoffel symbols for the same.

(b) State and prove Frenet - Serret theorem.

10 Attempt the following : **14**

(a) Prove in the usual notations :

$$\Gamma_{ij}^{l} = \frac{1}{2}\Sigma_{k=1}^{2} g^{kl}\left(\frac{\partial g_{ik}}{\partial u^{j}} + \frac{\partial g_{kj}}{\partial u^{i}} - \frac{\partial g_{ij}}{\partial u^{k}}\right)$$

(b) Prove that: A necessary and sufficient condition for a curve to be a straight line is that the curvature $K = 0$.
